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AUTHOR Lipson, Joseph I.
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ABSTRACT

This document describes the reasoning behind the: (a) selection and ordering of objectives, (b) choice of commercial materials, and (c) preparation of materials for an individually prescribed mathematics curriculum. General considerations and objective selection for individualization are reviewed for the selection of objectives. General considerations, variety and change of perspective, task complexity, student tolerance, effect of individualization, and changes for present operation are reviewed in the discussion concerning the ordering of objectives. The choice of commercial materials is discussed in relation to general considerations, considerations due to individualization, and present use. Finally, the preparation of materials is reviewed based on general and individualization considerations, updating principles, organization of materials, testing, and materials and testing used for individualization. (MJM)

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MATHEMATICS

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RATIONALE AND PHILOSOPHY OF REVISED (YEAR II) OAKLEAF MATHEMATICS CURRICULUM

Joseph I. Lipson

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Learning Research and Development Center
University of Pittsburgh

and

Research for Better Schools, Inc.
1700 Market Street
Philadelphia, Pennsylvania

SP008 156

RATIONALE AND PHILOSOPHY OF REVISED (YEAR II)
OAKLEAF MATHEMATICS CURRICULUM

Joseph I. Lipson

I will attempt to explain the reasoning behind the (1) selection of objectives, (2) ordering of objectives, (3) choice of commercial materials, and (4) preparation of materials by the teachers and the Center staff. The reasoning in each case will be divided into general considerations and special considerations arising from the fact of individualization. Finally, the organization of the materials and testing will be explained.

Selection of Objectives

General Considerations

The objectives were selected to conform to what I shall call "classical new math." The classical new math may be characterized as emphasizing and utilizing the logical development of arithmetic. In contrast, most classical arithmetic programs emphasized the learning of powerful computational algorithms based upon well memorized sets of tables. Newer mathematics programs aim at teaching the subject of mathematics in the spirit of unfolding the principles of arithmetic as abstracted from the properties of sets and involving the set properties of numbers. Our program strikes a middle course. In the logical development the laws of our number system (associativity, commutativity, distributivity, and the concept of inverse operations) are first developed through instances and then extensively applied (e.g., show $3 \cdot 9 = 27 = 3(3 + 6) = 9 + 18 = 27$. Use for $12 \times 12 = ?$: $(10 + 2)12 = 120 + 24 = 144$).

The algebraic laws are used to develop and reveal the basis of the algorithms (carrying, borrowing, multiplication, division, and operations with fractions) which are so important to computation (e.g., the progression from successive subtraction to long division). The long term objectives are still computational skills, but the hope is that the computational skill is founded upon a logical development which will enable the student to transfer to new problems and more complex mathematics more easily than would be the case if the tables were memorized and the algorithms were taught as rote rules.

The future algebra student is prepared through the introduction of frames, through the introduction of the concept of inverse operations and through the introduction of negative numbers. The future technician and scientist is prepared through the introduction of exponential notation, the use of the metric system, the development of operations with other number bases, as well as the use of negative numbers. At the same time that topics in the above areas are of use to future specialists, the insistence on all students mastering the objectives is justified on the ground that these important concepts refine the student's understanding of the more familiar computational concepts. For example, the concept of number itself is made clearer by familiarity and work with large and small numbers, with positive and negative numbers, by the use of exponential notation in place value charts, and by exercise in rounding and approximation.

One of the areas which the Oakleaf curriculum covers more than most curricula is that of geometric concepts. The reasoning here is simply that the concepts are important and that the student has the requisite behavior to learn them.

Our objective selection reflects the evidence that discriminating between instances of a concept is a powerful teaching technique. Suppes makes the important point that the most effective way to develop an arithmetic concept in a student (child or adult) is not through extensive practice in the use of a concept (e.g., addition), but through continually providing examples in which the student must decide what is and what is not an instance of the concept. The range and selection of these instances will determine how precisely the concept is defined in the mind of the student. To this end, key objectives require the student to choose the correct sign to complete an arithmetic statement, to determine which of two operations is greater than or less than another, to determine which operation is required to solve a problem, and other variations of these types of exercises.

Checking the reasonableness of an answer by approximate methods is a powerful method of reducing error rates and developing the ability for self-detection of errors. This method of checking by reasonableness is emphasized in our program in addition to more formal methods of checking answers by the use of inverse operations as well as the associative and commutative laws (adding or multiplying in a different order).

Objective Selection for Individualization

The principle requirement which individualization imposes upon the curriculum is the requirement that there must be alternate paths to the same objective, but that when a concept has been obtained the student will be able to display his mastery of a concept in a variety of settings. This variety of ways to reach a common objective can be explicitly stated by writing objectives which have the manner of reaching the objective clearly stated. For example, if the objective is to have the student add all two digit numbers, using a number line, using an abacus, or using the memorized addition tables with rules for carrying are different ways of exhibiting the objective. Materials can be provided so that the teacher assigns each method as it is needed, but the final objective would be tested without reference to the method used for solution. There are several risks to this procedure. Test writers will, in general, not write variant or transfer items for an objective unless this is specifically indicated in some way. Thus, if the objective states that the student must add all two digit numbers, the test writers usually will write items which embody the commonly accepted form of the problem, e.g., 27 + 39 = ? Test writers will not normally readon, "Well, if the student can add all two digit numbers he should be able to do the addition on a number line or an abacus, since these behaviors are, after all, just as much addition as the more conventional use of arabic numbers." On the other hand, a teacher will often avoid a method which either does not appeal to him or with which she is unfamiliar. Thus, the use of an abacus to addition may be avoided unless it is written as a separate objective, e.g., adds all two digit numbers using an abacus.

Making alternate paths to an objective appear as clearly stated parts of the written objectives has many advantages. Test writing for transfer and teaching procedure alternatives become more neatly defined operations. The manner in which a concept is being defined is more clearly visible to an outside observer and thus, the system is more open to feedback and corrective change.

It might appear that the curriculum is open to excessive numbers of objectives. However, not all alternate paths in existence are written into the curriculum. Only those methods which have given evidence of either teaching efficiency or usefulness to some later objective are used. Thus the use of the number line not only serves as a method of teaching

addition, it is an extremely important forerunner of analytic geometry and vectors. In addition to the restriction of the numbers of objectives in the curriculum, the curriculum is rendered manageable because students can pretest out of any objective for which they show mastery. It is argued then, that once a student acquires a concept through mastery of a variety of objectives, he will begin to test out of objectives which are merely variants of, or extensions of the concept he has achieved.

To summarize, an individualized program requires alternate paths to a major conceptual objective since not all students learn equally well by the same approach. For example, some students may need more practice, some students may need other instances of a concept (e.g., dividing actual arrays of objects to illustrate the concept of division. In turn, the curriculum can stand the additional objectives without becoming too long because students who have a major concept under control can test out of many related objectives.

Ordering of Objectives

General Considerations

Objectives are ordered to: (1) insure that the entering behavior of the student is adequate to the learning task, (2) to provide variety and change of perspective among the topics of arithmetic, and (3) to match the level of task complexity which the student can tolerate, i.e., as definitions, constructions, and examples become more complex, an objective involving such complexity may be delayed even though a student may formally have the necessary entering behavior.

To elaborate on this ordering problem, it is clear that in arithmetic there is a hierarchy of tasks such that subsequent tasks depend upon earlier behaviors. Our curriculum merely tries to identify and record this ligical progression of tasks. Occasionally an objective is found to be difficult for the students to achieve. There are two approaches to this problem. One approach is to develop new instructional materials based upon the actual (rather than the improperly assumed) entering behavior of the students. This is the approach used to keep flexibility in the program without rewriting the ordering of the objectives each time a trouble spot is found. However, when the curriculum is being revised, as will be discussed when we discuss changes for this year, the opportunity is presented to change the position of a trouble-

some objective. When this is done, new teaching approaches may still be required to utilize the changed entering repertoire of the students.

The point should be made that there are two important considerations in ordering the topics of arithmetic. One consideration is what important topics can be efficiently taught with the student's entering behavior and the other consideration is which of these behaviors is most profitable to future learning. Since there is always a surplus of behaviors which can be taught, the ordering problem is reduced to trying to evaluate which behaviors are most useful to later learning. On this basis it is often pointless to hammer at an isolated topic which is causing difficulty. It is usually more profitable to re-evaluate the sequence and move the troublesome topic back so that the student encounters the topic when he has more related concepts to bring to bear on the difficult problem. For example, counting in other number bases than base ten can be taught at almost any point in the curriculum. If it is effectively taught, it helps the student to understand the process of numeration in general and the base ten system in particular. If it is causing the student or the class difficulty, it would seem to be more profitable to delay the introduction of counting in other number bases until the student has more experience in his natural (base ten) system.

Variety and Change of Perspective

It is conceivable that once a student started to learn to count that he could learn counting operations to extremely large numbers (millions, billions). The Oakleaf curriculum takes the position that counting operations should be taught as the numbers are needed for other operations, i.e., the numbers taught should be (a) related to the numbers of objects that the student's environment provides, and (b) useful to the operations of addition, subtraction, multiplication, and division which are to be taught. When sufficient numeration to the subsequent tasks has been developed, the instructional unit shifts to another area of arithmetic. Of course, the reasoning is somewhat circular here. Once the limits have been placed upon the numeration unit, this sets limits upon the operations to be performed. In the final analysis, some reliance is placed upon tradition in the absence of clear evidence or theoretical reasons to the contrary.

Task Complexity and Student Tolerance

Although much is made of the fact that almost anything can be meaningfully taught at some level to almost anyone, the essence of certain topics is sophisticated, subtle, and complex. While aspects of such topics can be taught at earlier stages of education and certainly we can prepare the student with appropriate preliminary objectives, the methods of teaching of many objectives such as the division of fractions, decimals, and square roots require many behaviors which are neither well identified nor well developed in the very young student. For example, the ability to hold in mind a sequence of instructions may limit the teaching of an otherwise simple conceptual problem.

Effect of Individualization

The main idea of individualization is that it makes the problem of ordering the curriculum less critical than it might seem. There are two ways that the individualized program mitigates an error in the order of objectives. By an error in the order of objectives is meant that a student is asked to perform tasks which are inordinately difficult for him because he does not have the required entering behavior. The inadequate entering behavior may be an isolated case with a single student or it may be a factor with a large group.

One way to remedy a defect in ordering is simply for the teacher to assign units in a different order. There is no way to stop a teacher from doing this and there is no evidence that they have abused this option. If anything, I would judge that they have not used this option with enough flair and imagination.

Another way to rectify an ordering error in an individualized program is to have different sets of instructional materials for students of different entering behaviors. A simple example would be additonal work pages with big spaces and big numerals and simple instructions for young students and small numerals and spaces and more sophisticated instructions for older students. This point will be returned to in the discussion of materials.

Changes for Present Operation

Extensive changes in the ordering of objectives were made at the upper levels of the mathematics program based upon the experience of the

first year at Oakleaf. The main type of evidence used was that some objectives were proving too difficult at the stage they were assigned; other objectives were not only easy at the stage they were assigned, but it appeared that earlier introduction would facilitate later learning. An example of this can be found in the fractions sequence. Delay of the introduction of greatest common denominator and least common multiple delayed the development of other concepts using fractions. Therefore, these objectives were moved down. It remains to be seen whether the hoped for benefits will be realized.

An interesting point is that changes in objectives were independently proposed by a group from the Center and by the teachers. It is, I believe, a hopeful sign that the proposals of the two groups were in agreement on all major points and were in agreement 85% to 90% of the time on all objective changes.

Choice of Commercial Materials

General Considerations

Workbook materials were chosen to represent a spectrum of approaches consistent with our attempt to have a new mathematics curriculum. It was held to be a positive good to have several different formats, styles, and approaches to the presentation of arithmetic problems. It is felt that the variety presented by several commercial programs in addition to teacher written pages would assist the children in generalizing to new situations both within and outside the school setting.

The materials were chosen by a committee consisting of educators, teachers, mathematicians, and in consultation with educational psychologists. Still, the materials were chosen on relatively inadequate grounds of reputation, personal liking, and information provided by the publisher. Only after the materials were assigned to our curriculum and we had used them for a year did we have the kind of information we should have had when making the initial choices. This reflects the fact that curriculum development is still in its infancy. Thus without any adequate tools of analyzing a commercial product, one sets up a committee and hopes for the best. After assignment to our curriculum, we can say a great deal about the strengths and weaknesses of each of the commercial programs. Some topics are simply not adequately dealt with. The reverse was also true. Apparently because no task and work analysis had been done, some types

of problems are provided in numbers far beyond the needs of instruction and/or practice.

Considerations Due to Individualization

Individualization was one of the principle reasons for using seven different sets of commercial materials. It was felt that the work assignments might require different approaches for different students. For example, the "counting man" is a help for some students and confusing for others. Of course, as discussed above, any particular approach could be insured by incorporating it into the objectives. This could never be carried out completely since every behavioral objective will inspire different teaching approaches among different writers. It was expected that some approaches would become dominant by a sort of natural selection of the classroom. Each teacher would tend to assign work pages which he liked and which proved successful for him. This would be reflected in the numbers of each page used. The difficulty in this popularity contest is that it is difficult to evaluate how much of assignment selection of pages is due to intrinsic worth and how much is due to prejudice in one form or another or perhaps the result of poor page assignment, i.e., a page may be a terrific lesson if assigned to numeration, but a poor page in trying to develop a behavior in place value. If a unit is incorrectly assigned to a behavior it may suffer extinction which it does not deserve. Some attempt to avoid this was made by having two people independently assign each unit and any discrepancies were discussed and reconciled. At any rate, it should continue to be true that in the interests of individualized instruction a variety of approaches for teaching a behavior must be available to the teacher. It seems that the form of this variety might be better handled in writing special objectives which require major teaching methods, while more subtle differences in approach will be handled by having different types of problems and lesson formats. Once the curriculum is more firmly grounded, the shotgun approach to assuring variety should no longer be needed.

Present Use

A page use analysis was made and pages which were essentially unused were tabulated. When a commercial workbook had a majority of pages in the relatively unused category, the workbook was not bought for year

two and the pages dropped from the materials. On the other hand, since students were expected to be moving into higher levels than last year, new books were bought for assignment to the upper levels of work.

Preparation of Materials by Teachers and Center Staff

General Considerations

In many cases there was simply no commercial material to assign to a given objective. In other cases, the materials were skimpy or ineffective. In still other instances, the materials appeared to be adequate only to fail in the classroom. In each of these cases the resources of the Center were used in conjunction with the teachers. The procedure during the school year is that the teaching approach is tried out by direct teaching while the work materials are being written. Once the materials are written in rough form they are tried on a few students to check for bugs. Finally, the pages are run off, assigned cart numbers and made available for assignment by the teachers. Follow-up consists of checking posttest scores of the students both before and after the new pages have been made available. If the school is not in session, pages are written with a simple editorial review. If, however, the pages do not work at the school, we always have the same flexibility of revision as before.

Individualization Considerations

It is important to note the time for revision and provision of new materials from the time a trouble spot occurs.

In the Oakleaf project the work pages are stored for use as individual pages or in lesson groups of pages. As a result, different lesson pages can be assigned to different students. Thus, it is crucial that teachers prescribe well. As a result, when additional pages are needed or old pages prove useless, changes can be instituted without changing the whole set of materials as would be the case if workbooks were used. The time required to institute a fairly extensive change (e.g., a major change in the method of teaching addition of fractions) is about one month. This relatively short time constant for revision is to be compared to a minimum of a year for any commercial workbook. Also to be noted is that a unit or page can be revised which would not in general

be considered important enough to cause the revision of an entire workbook. In summary, the needs of individualized instruction have incidentally given us an instrument with powerful corrective features which can be changed swiftly enough to follow changes in teaching methods and changes in the character of the student body. Rapid and continuous revision of the materials used for instruction is not an objective to be lightly ignored.

A requirement of individualization is that the teacher and the student be able to quickly infer the directions for completing the work on a page. There is little or no time available for referring to the teacher's handbook for information. In a normal class, the entire class would be given working directions for a page of problems. In an individualizaed program this is not possible.

Up-dating Principles

In order to deal with the above requirements, extensive changes in materials were made by the teachers and Center staff. Each commercial page was examined for the adequacy of its working directions. When these were inadequate, directions were written and duplicated on the commercial pages.

Extensive sections of pages were written when adequate commerical pages were either absent or of poor teaching quality. The criterion to describe poor teaching quality is that the students were unable to master the posttests or that they took longer than was thought reasonable to achieve mastery. As indicated in our basic curriculum approach, special attention was paid to writing materials which lead the student from basic numeration concepts to algorithms (e.g., sections were written for carrying in addition, borrowing in subtraction, the multiplication algorithm and the long division algorithm).

Since books for which less than half the pages were not re-ordered, pages were written to replace those pages which had been extensively used.

Organization of Materials

The materials are arranged in two forms. One form is by objective, the other is by individual page or lesson unit for distribution to the children. The materials for each objective are put together in one folder. The collection of such folders is called the master file which is used for

work prescription. Each page has two numbers on it. The pages for student use are stored on six divided library carts. Each page has a cart location number. The pages are then stored on carts by this number. Each cart has a capacity of about 600 different pages with 30 to 50 duplicate copies of each page. In order to provide cross-indexing, a list of pages is made with the source and behavioral assignment of each page; a set of master workbooks of the original materials is kept which identifies the behavioral assignment of each page. Papers written by local people have the behavioral assignment printed on each page. The master file, then, makes the materials available for the teacher in writing prescriptions and in gaining an overall view of the materials for teaching an objective. The carts supply a method for dispensing the assigned pages to the students. The master workbooks and page lists are cross-reference lists in case a page is lost from a master file.

Testing

Testing utilizes three kinds of tests: placement tests, pre- and posttests for each unit, and curriculum embedded tests. The placement tests are rough scale tests to determine the general placement of the child in the curriculum stream. The unit tests are mastery tests which the student must pass before moving on to the next unit. The curriculum embedded tests are short quizzes which are placed on the carts which are useful to quickly determine whether a student is meeting the challenge of the work from day to day.

Materials and Testing for Individualization

The materials must be organized so that different children get different work assignments on their path to meet the common objectives. It is part of the philosophy of the program that no child's life should be biased by giving different students different objectives. The differences in achievement will principally lie in how far each student can go in the curriculum during the elementary school years. Thus, the materials are broken down to the smallest practical unit to permit maximum flexibility in work assignments. The tests are critical in ensuring that no student is required to work in an area which he has already mastered. In addition, they (the tests) are a powerful diagnostic aid to the teacher in providing work prescriptions for the individual students. The philosophy is to provide a maximum of information to the

teacher so that her decisions on educational procedure can fit the needs of the individual student in the two minutes per student per day which the teacher has.

Current Procedures

Alternate forms of the unit tests were written so that the student did not repeat the same test as a posttest after taking it as a pretest. Placement tests were reduced in number at the same time that the questions were analyzed more closely for their use as diagnostic items for placement. Finally, whereas in Year One we tried to use commercial pages as curriculum embedded tests, these were so obviously poor that a complete set of curriculum embedded tests were written.

In Year One, clerks were used to grading all work. In Year Two, self-grading of workpages is planned. To this end, answer keys are placed with each page on the carts. The answer keys are encased in plastic protectors. Tests, however, will continue to be graded and recorded by the clerks; this will also be true of the curriculum embedded tests.

Summary

The philosophy of the current Oakleaf Mathematics Program is to provide computational skill, use of basic laws of arithmetic in developing the operations and properties of the number system. The limitations on the use of the new math were based upon the backgrouns of the teachers and the availability of materials which would fit our classroom design. Further, the committee which was called to initiate the math program was hesitant to go all the way with any single new math program. Multiple commercial materials were used to provide multiple teaching approaches and multiple ways of presenting the same problem. Multiple teaching approaches were desired in the interests of individualization while multiple problem formats were desired to assist the students to generalize. Changes were made in the curriculum, materials, and tests on the basis of our experience from the first year. A point of emphasis is the bridging by logical steps from basis rules of arithmetic to algorithms.